Monday 16 June 2014 – Morning

A2 GCE MATHEMATICS

4735/01 Probability & Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4735/01
- List of Formulae (MF1)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Answer **all** the questions.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Give non-exact numerical answers correct to 3 significant figures unless a different degree of accuracy is specified in the question or is clearly appropriate.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.

- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- **You are reminded of the need for clear presentation in your answers.**
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

1 A teacher believes that the calculator paper in a GCSE Mathematics examination was easier than the non-calculator paper. The marks of a random sample of ten students are shown in the table.

Student	A	B	C	D	E	F	G	H	I	J
Mark on paper 1 (non-calculator)	66	79	58	87	67	55	75	62	50	84
Mark on paper 2 (calculator)	57	84	70	90	75	42	82	72	65	82

(i) Use a Wilcoxon signed-rank test, at the 5% significance level, to test the teacher's belief. **[7]**

(ii) State the assumption necessary for this test to be applied. **[1]**

2 During an outbreak of a disease, it is known that 68% of people do not have the disease. Of people with the disease, 96% react positively to a test for diagnosing it, as do m% of people who do not have the disease.

(i) In the case $m = 8$, find the probability that a randomly chosen person has the disease, given that the person reacts positively to the test. **[5]**

(ii) What value of m would be required for the answer to part **(i)** to be 0.95? **[4]**

3 The discrete random variable X has probability generating function $\frac{t}{a-bt}$, where a and b are constants.

(i) Find a relationship between a and b. **[1]**

(ii) Use the probability generating function to find $\mathrm{E}(X)$ in terms of a, giving your answer as simply as possible. **[3]**

(iii) Expand the probability generating function as a power series, as far as the term in t^3, giving the coefficients in terms of a and b. **[3]**

(iv) Name the distribution for which $\frac{t}{a-bt}$ is the probability generating function, and state its parameter(s) in terms of a. **[2]**

4 The continuous random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} x & 0 \leqslant x \leqslant 1, \\ 2-x & 1 \leqslant x \leqslant 2, \\ 0 & \text{otherwise.} \end{cases}$$

(i) Show that the moment generating function of X is $\frac{(\mathrm{e}^t-1)^2}{t^2}$. **[6]**

Y_1 and Y_2 are independent observations of a random variable Y. The moment generating function of $Y_1 + Y_2$ is $\frac{(\mathrm{e}^t-1)^2}{t^2}$.

(ii) Write down the moment generating function of Y. **[1]**

(iii) Use the expansion of e^t to find $\mathrm{Var}(Y)$. **[5]**

(iv) Deduce the value of $\mathrm{Var}(X)$. **[1]**

5 Two discrete random variables X and Y have a joint probability distribution defined by

$$P(X = x, Y = y) = a(x + y + 1) \qquad \text{for } x = 0, 1, 2 \text{ and } y = 0, 1, 2,$$

where a is a constant.

(i) Show that $a = \frac{1}{27}$. **[2]**

(ii) Find $E(X)$. **[2]**

(iii) Find $\mathrm{Cov}(X, Y)$. **[5]**

(iv) Are X and Y independent? Give a reason for your answer. **[2]**

(v) Find $P(X = 1 | Y = 2)$. **[2]**

6 A Wilcoxon rank-sum test with samples of sizes 11 and 12 is carried out.

(i) What is the least possible value of the test statistic W? **[2]**

(ii) The null hypothesis is that the two samples came from identical populations. Given that the null hypothesis was rejected at the 1% level using a 2-tail test, find the set of possible values of W. **[6]**

7 The continuous random variable X has probability density function

$$f(x) = \begin{cases} \dfrac{k}{(x+\theta)^5} & \text{for } x \geqslant 0, \\ 0 & \text{otherwise,} \end{cases}$$

where k is a positive constant and θ is a parameter taking positive values.

(i) Find an expression for k in terms of θ. **[2]**

(ii) Show that $E(X) = \frac{1}{3}\theta$. **[3]**

You are given that $\mathrm{Var}(X) = \frac{2}{9}\theta^2$. A random sample $X_1, X_2, \ldots, X_n$ of n observations of X is obtained. The estimator T_1 is defined as $T_1 = \dfrac{3}{n}\displaystyle\sum_{i=1}^{n} X_i$.

(iii) Show that T_1 is an unbiased estimator of θ, and find the variance of T_1. **[3]**

(iv) A second unbiased estimator T_2 is defined by $T_2 = \frac{1}{3}(X_1 + 3X_2 + 5X_3)$. For the case $n = 3$, which of T_1 and T_2 is more efficient? **[4]**

END OF QUESTION PAPER

<table>
<tr><th colspan="3">Question</th><th>Answer</th><th>Marks</th><th colspan="2">Guidance</th></tr>
<tr><td>1</td><td>(i)</td><td></td><td>$H_0: m_1 = m_2$ $H_1: m_2 > m_1$
<table><tr><td>–</td><td>+</td><td>+</td><td>+</td><td>+</td><td>–</td><td>+</td><td>+</td><td>+</td><td>–</td></tr><tr><td>9</td><td>5</td><td>12</td><td>3</td><td>8</td><td>13</td><td>7</td><td>10</td><td>15</td><td>2</td></tr><tr><td>6</td><td>3</td><td>8</td><td>2</td><td>5</td><td>9</td><td>4</td><td>7</td><td>10</td><td>1</td></tr></table>$T^+ = 39$; $T^- = 16$; $T = 16$
CV = 10
TS > CV, do not reject H_0
Insufficient evidence that the calculator paper was easier. oe</td><td>B1
M1,A1
A1
B1
M1
A1
[7]</td><td>Allow equiv hyps using differences.
If in words, needs ‘population’
1st A1 is for correct differences.
2nd A1 is for correct T from correct ranks.
ft TS, CV
ft TS
Contextualised, not over-assertive.</td><td>NOT: marks ..... NOT papers.......
NOT: mean
NOT: difference, unless clearly 2-tail</td></tr>
<tr><td></td><td>(ii)</td><td></td><td>Differences symmetrical</td><td>B1
[1]</td><td></td><td>.</td></tr>
<tr><td>2</td><td>(i)</td><td></td><td>0.32×0.96 or 0.68×0.08
Both, added.
= 0.3616
$0.32 \times 0.96 \div "0.3616"$
0.850</td><td>M1
M1
A1
M1
A1
[5]</td><td>May be implied.
Allow 0.85 or $\frac{96}{113}$</td><td>Allow M marks for 0.8 instead of 0.08 or incorrect 1-0.68.</td></tr>
<tr><td></td><td>(ii)</td><td></td><td>$\frac{0.32 \times 0.96}{0.32 \times 0.96 + 0.68 \times p} = 0.95$
Solve
$p = 0.0238$, so $m = 2.38$</td><td>M1,A1
M1
A1
[4]</td><td>Allow 0.3072
Allow failure to multiply brackets correctly, but NOT divide instead of subtract or vv.
$\frac{192}{1075}$</td><td></td></tr>
</table>

Question			Answer	Marks	Guidance	
3	(i)		$a-b=1$ oe isw this part.	B1 **[1]**	Allow $\frac{1}{a-b}=1$	Use $G_X(1)=1$
	(ii)		$\frac{(a-bt+bt)}{(a-bt)^2}$ Use $G'_X(1)=1$ a	M1 M1 A1 **[3]**	Use quotient or product rule.	NOT with $\frac{t}{a-at}$
	(iii)		Binomial expansion $\frac{t}{a}+\frac{bt^2}{a^2}+\frac{b^2t^3}{a^3}$	M1 A2 **[3]**	Ignore errors in setting up ...(1+...)^(-1)for M1 A1 for 2 terms correct.	Or t(1-b+b²)+t²(b-2b²)+b²t³
	(iv)		Geo($\frac{1}{a}$)	B1,B1ft **[2]**	$\frac{1}{\text{(ii)}}$ ft B1	
4	(i)		$\int_0^1 xe^{tx}dx+\int_1^2(2-x)e^{tx}dx$ $\left[\frac{xe^{tx}}{t}\right]+\left[\frac{-e^{tx}}{t^2}\right],\left[\frac{(2-x)e^{tx}}{t}\right]+\left[\frac{e^{tx}}{t^2}\right]$ $\frac{e^t}{t}-\frac{e^t}{t^2}+\frac{1}{t^2}$ $+\frac{e^{2t}}{t^2}-\frac{e^t}{t^2}-\frac{e^t}{t}$ $\frac{(e^t-1)^2}{t^2}$ AG	M1 M1 A1 A2 A1	 Int by parts, either integral. A1 for one error in 2nd integral. cwo	 No need for limits for this mark.

Question			Answer	Marks	Guidance	
				[6]		
	(ii)		$\frac{e^t - 1}{t}$	B1 **[1]**		
	(iii)		$\frac{1}{t}\left(t+\frac{t^2}{2}+\frac{t^3}{6}+\frac{t^4}{24}+...\right)$ $1+\frac{t}{2}+\frac{t^2}{6}$ $E(Y)=\frac{1}{2}$, $E(Y^2)=\frac{1}{3}$ $Var(Y) = \frac{1}{3}-\frac{1}{4}$ $= \frac{1}{12}$	M1 A1 B1ft M1 A1 **[5]**	Need attempt at first 3 terms in brackets. Allow use of ! For both, ft coeff of t, 2× coeff of t^2	
	(iv)		$\frac{1}{6}$	B1ft **[1]**	ft 2Var(Y)	
5	**(i)**		$a, 2a, 3a; 2a, 3a, 4a; 3a, 4a, 5a$ $a=\frac{1}{27}$ AG	B1 B1 **[2]**	Allow a(0+0+1), etc Must see (27a)oe=1	
	(ii)		<table><tr><td>x</td><td>0</td><td>1</td><td>2</td></tr><tr><td>p</td><td>$\frac{2}{9}$</td><td>$\frac{3}{9}$</td><td>$\frac{4}{9}$</td></tr></table> $E(X) = \frac{11}{9}$	B1 B1 **[2]**	oe	
	(iii)		<table><tr><td>xy</td><td>0</td><td>1</td><td>2</td><td>4</td></tr><tr><td>p</td><td>$\frac{11}{27}$</td><td>$\frac{3}{27}$</td><td>$\frac{8}{27}$</td><td>$\frac{5}{27}$</td></tr></table> $E(XY) = \frac{39}{27}$, $E(Y) = \frac{11}{9}$ ft Use $Cov(XY) = E(XY) - E(X)E(Y)$	B1 B1,B1ft M1	Seen or implied eg by 3a+8a+8a+20a $E(XY)=\frac{13}{9}$	

Question			Answer	Marks	Guidance	
			$-\frac{4}{81}$	A1 **[5]**		
	(iv)		Cov $\neq 0$ No	M1 A1 **[2]**	ft non-zero Cov.	OR eg P(0,0)≠P(0)xP(0)
	(v)		$\frac{4}{27} \div \frac{12}{27}$ $= \frac{1}{3}$	M1 A1 **[2]**	4a/12a	
6	**(i)**		$1 + 2 + \ldots + 11$ $= 66$	M1 A1 **[2]**	M0 if followed by incorrect work.	
	(ii)		(N) (132,264) $\frac{(W + 0.5 - "132")}{\sqrt{"264"}}$ $< -$ 2.576 Solve inequality $< 89.6 \quad (66 \le)\ W \le 89$	B1 M1 M1* B1 *M1 A1 **[6]**	Allow wrong, or no, cc. May be earned later. Allow 2.58 or equation if final answer uses < or ≤ Integer needed.	Allow reversed if consistent OR 132(-0.5)±z×√264 M1 z=2.576 or 2.58 B1 (89.6,[173.4]) A1 < lower limit M1 ≤89 A1 Allow if lower limit only considered.
7	**(i)**		$\int_0^\infty k(x+\theta)^{-5}\mathrm{d}x = 1$ $k = 4\theta^4$	M1 A1 **[2]**		
	(ii)		$\int_0^\infty 4\theta^4 x(x+\theta)^{-5}\,\mathrm{d}x$ Attempt int. by parts or sub'n. $= \frac{\theta}{3}\ AG$	M1 M1 A1 **[3]**	ft k.	

Question			Answer	Marks	Guidance	
	(iii)		$\frac{3}{n}\sum_{1}^{n} X_i = 3\mathrm{E}(X) = 3(\frac{\theta}{3}) = \theta$	B1	or $\frac{3}{n} \times n \times \frac{\theta}{3}$	
			$\mathrm{Var}(T_1) = 9\Sigma\mathrm{Var}(X)/n^2$	M1	or $\frac{9}{n^2} \times n \times \frac{2\theta^2}{9}$	
			$= \frac{2\theta^2}{n}$	A1		
				[3]		
	(iv)		$\mathrm{Var}(T_2) = \frac{1}{9}\mathrm{Var}(X_1) + \mathrm{Var}(X_2) + \frac{25}{9}\mathrm{Var}(X_3)$	M1	Allow $\mathrm{Var}(T_1) = 3\sigma^2$ and $\mathrm{Var}(T_2) = \frac{35\sigma^2}{9}$	
			$= \frac{70\theta^2}{81}$	A1		
			$> \frac{2\theta^2}{3}$	M1	ft their Vars.	
			T_1 is more efficient	A1		
				[4]		